# Radio emission processes - Part I

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## 1 Introduction

Radio emission from the Sun is produced incoherently or coherently. Incoherent emission consists of free - free emission (Bremsstrahlung) and Gyro - emission. Plasma emission and Electron cyclotron maser emission are the main radio emission processes in the case of coherent radio emission. Incoherent emission results from continuum processes, such as thermal particle distribution, that produce through Columb collisions or thermal/mildly relativistic electron distribution that produce through gyroresonance/gyrosynchrotron emission. Coherent emission is produced by kinetic instabilities from unstable particle distribution. No spectral lines in emission or absorption due to atomic/molecular transitions have been discovered in the radio frequency band on the Sun.

## 2 Bremsstrahlung

The term bremsstrahlung means braking radiation in German and is the electromagnetic radiation produced by the acceleration of a charged particle like electron in the Coulomb field of ambient ions. In a fully ionized plasma, electrons and ions move freely interacting with each other through their electrostatic charges. The most important interaction is the scattering of a free electron in the Coulomb field of an ion. Since the electron and the ion are free particles before and after the interaction, the radiation emitted is called as free - free radiation. A photon is emitted with an energy corresponding to the difference of the outgoing to the incoming kinetic energy of the electron according to the principle of conservation of energy. Figure 1 shows an example of a binary collision between a electron of charge + e and velocity v with an ion of charge $Z_i$. The effect of this collision is to deflect the incoming electron by an amount which depends on the impact parameter b. Due to the small angle collisions, the trajectory of the incoming electron is determined by a multitude of small deviations. The bremsstrahlung emission from a single electron is calculated using the equations given in Rybiciki and Lightman (1979). It is assumed that the electron energy in the intial stage is greater than energy lost by the electron when it interact with the ion. For radio waves, distant encounters of electrons with ions called small angle deflections are more important than the relatively close encounters and large deflections. When an electron

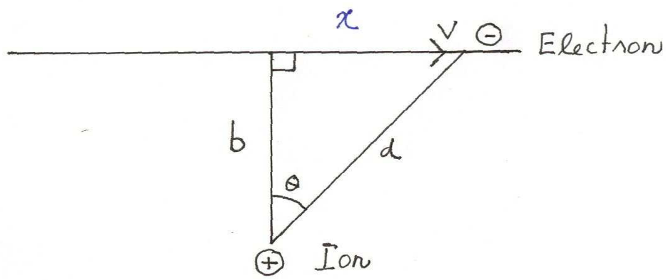


**Fig. 1.** An electron passing by a heavy ion. Due to weak scattering, low energy radio photons are produced. The distance of closest approach is called the impact barameter b. The distance between the ion and the electron is d and $\theta$ is the angle betwen b and d.

of charge e passes by an atom having Z ions, due to the electrostatic force, the electron is accelerated both parallel and perpendicular to the direction of electron's velocity. Emission due to acceleration in the perpendicular direction is in the radio band. The force in the perpendicular direction is given by

$$F_{\perp} = m_e a_{\perp} = \frac{Ze^2 cos(\theta)}{d^2} = \frac{Ze^2 cos^3(\theta)}{b^2} \tag{1}$$

where d is the distance between the electron and the ion, b is the impact parameter and $m_e$ is the mass of the electron. According to Larmor's formula, the total power radiated by the electron is $\propto a^2$ and is given by

$$P = \frac{2}{3}\frac{e^2 a^2}{c^3} \tag{2}$$

where a is the acceleration, e is the electric charge, and c is the speed of light. The total energy radiated by the electron in time dt is

$$W = \int_{\infty}^{\infty} P dt \tag{3}$$

This energy is emitted in a single pulse of duration $\tau = \frac{b}{v}$ , so that the power spectrum is flat over all frequencies upto a maximum frequency $\nu_{max} = \frac{1}{2\pi\tau}$ $= \frac{v}{2\pi b}$ and falls of rapidly at high frequencies. Assuming the power spectrum is flat up to frequency $\nu = \frac{v}{2\pi b}$, we get the average energy per unit frequency emitted by a single electron as $W_\nu = \frac{W}{\nu_{max}}$. Therefore $W_\nu$ is given by

$$\frac{\pi^2}{2} \frac{Z^2 e^6}{c^3 m_e^2} \left[ \frac{1}{b^2 v^2} \right] \tag{4}$$

In an ensemble of particles, the number of electrons passing any ion per unit time with impact parameter b to b+db and speed ranging from v to v+dv is $N_e(2\pi b db) v f(v) dv$ where f(v) is the normalised speed distribution of electrons. The number N(v,b) of such encounters per unit volume /unit time is given by $2\pi b db (v f(v) dv) N_e N_i$. where $N_e \& N_i$ are the electron and ion number densities. The isotropic spectral power emitted at frequency $\nu$ is given by 4 $\pi\epsilon_\nu$ where $\epsilon_\nu$ is the emission coefficient.

$$4\pi\epsilon_\nu = \int_{b=0}^{\infty} \int_{v=0}^{\infty} W_\nu(v,b) N(v,B) dv db \tag{5}$$

With the derived value of $W_\nu$ the above equation becomes

$$\frac{\pi^3 Z^2 e^6 N_e N_i}{c^3 m_e^2} \int_{v=0}^{\infty} \frac{f(v)}{v} dv \int_{b=0}^{\infty} \frac{db}{b} \tag{6}$$

In the above equation the term $\int_{b=0}^{\infty} \frac{db}{b}$ diverges. Therefore let us consider the limits of b from $b_{min}$ to $b_{max}$ . It is convenient to write $ln\left(\frac{b_{min}}{b_{max}}\right)$ as $G_{ff}(v,\nu)$ which is called as Gaunt factor ( Karzas and Latter, 1961). In the case of a non relativistic Maxwellian velocity distribution. The emission coefficient is

$$\epsilon_\nu = \frac{\pi^2 Z^2 e^6 N_e Ni}{4c^3 m_e^2} \left(\frac{2m_e}{\pi k T}\right)^{3/2} ln\left(\frac{b_{max}}{b_{min}}\right) \tag{7}$$

Assuming Local thermo dynamic equilibrium (LTE) at temperature T the absorption coefficient $k_\nu$ can be related to the emission coefficient and the brightness by the Kirchoff's law by

$$k_\nu = \frac{e_\nu}{B_\nu(T)} = \frac{\epsilon_\nu c^2}{2kT\nu^2} \tag{8}$$

The emission coefficient is therefore

$$k_\nu = \frac{1}{\nu^2 T^{3/2}} \left[\frac{Z^2 e^6 N_e N_i}{c}\right] \left[\frac{1}{\sqrt{2\pi m_e^3 k^3}}\right] \frac{\pi^2}{4} ln\left(\frac{b_{max}}{b_{min}}\right) \tag{9}$$

The properties of the radiation field is modified when it propagates through a medium to the observer and is described by the radiative transfer equation (Kraus (1966).

$$\frac{dI_\nu}{ds} = \eta_\nu - k_\nu I_\nu \tag{10}$$

The term $\eta_\nu$ is a source term while the term $k_\nu I_\nu$ is an absorption term. The later is used to define the optical depth along the line of sight. ie : $\tau_\nu = \int_0^s k_\nu ds$
The solution of equation 10 is of the form

$$I_\nu = I_{\nu(0)} e^{-\tau_c} + \frac{\eta_\nu}{k_\nu}(1 - e^{-\tau_c}) \tag{11}$$

In the case of a plasma cloud sitting between the source and the observer, the observed brightness is the combination of the source and contribution from the plasma cloud. Expressing the intensity in terms of temperature, we get

$$T_b = T_s e^{-\tau_c} + T_c(1 - e^{-\tau_c}) \tag{12}$$

The optical depth in the case of free - free emission is

$$\tau_\nu = \int -k_\nu ds \propto \int \frac{N_e N_i}{\nu^2 T^3/2} ds \approx \int \frac{N_e^2}{\nu^2 T^3/2} ds \tag{13}$$

From the observed brightness temperature the flux density can be calculated using the equation

$$S = \frac{2k_B}{\lambda^2} \int T_b d\Omega \tag{14}$$

where $k_B$ is the Boltzmann's constant, $\lambda$ is the wavelength and $T_b$ is the brightness temperature and $d\Omega$ is the size of the source. Since the absortion coefficient scales with square of the density, the free - free emission is dominant in regions of high density.

## 3 Gyro-emission

When a non-relativistic electron gyrates about the magnetic field, cyclotron radiation is emitted at the frequency of gyration called the gyrofrequency and is given by $f_B = eB/2\pi m_e c = 2.8B$ MHz where B is in Gauss. The electron gyrofrequency is independent of velocity and therefore independent of the kinetic energy of the electron ( $E = \frac{1}{2} m_e v^2$ when $v << c$ ). Since the gyrofrequency is single valued it really represents monochromatic or line radiation rather than continuum radiation. For low velocity electrons ($v < .03c$ ), only a single gyrofrequency emission occur. Gyroresonance emission involves a resonance between the electromagnetic waves and electron spiralling along the magnetic field lines at the electron gyrofrequency. This resonance produces strong coupling between the electrons and radiation at low harmonics of the gyrofrequency ($f = s f_B$ (s=1, 2, 3 .....)). At mildly relativistic speeds ( electron energies (100- 300 kev)), 10 - 100 harmonics of the cyclotron frquency occur, which overlap to produce a continuous spectrum. This form of emission is called gyrosynchrotron emission At highly relativistic speeds, the emission occurs at many harmonics so close together that look like a continuum emission. This type of emission is called synchrotron emission.

## 4 Synchrotron emission

The general Larmor's equation is a nonrelativistic one and and are correct only in the inertial frames where the electron velocity v < < c. Using the Lorentz transform, the results can be transformed into other frames. We now calculate the total power radiated by a ultra relativistic electron in a mgnetic field parallel to the x -axis. The primed coordinates are used to describe an inertial frame in which the electron is nearly at rest. The Larmor equation in this case is given by

$$P' = \frac{2e^2 \left(a'_\perp\right)^2}{3c^3} \tag{15}$$

where $a\prime_\perp$ is the magnetic acceleration in the observer's frame. We have

$$a_\perp = \frac{a'_\perp}{\gamma^2} \tag{16}$$

The Larmor equation now can be written as

$$P' = \frac{2e^2}{3c^3} \left(a'_\perp\right)^2 = \frac{2e^2 a_\perp^2 \gamma^4}{3c^3} \tag{17}$$

Since $a_{\perp} = \omega v_{\perp}$ and $\omega = \dfrac{eB}{\gamma mc}$ in the relativistic case.
The time averagad power radiated by an electron is given by

$$P = \frac{2e^2}{3c^3}\gamma^2 \frac{e^2 B^2 v^2 sin^2\theta}{m^2 c^2} \tag{18}$$

where $\theta$ is the angle betwen the **v** and **B** and is called the pitch angle.
The above power can be expressed in terms of $\sigma_T$, Thomson cross section of an electron ( $\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2$ ) and the magnetic energy density ($U_B = \frac{B^2}{8\pi}$)
The power radiated by a single electron is therefore

$$P = \left[\frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2\right] 2\left(\frac{B^2}{8\pi}\right) c\gamma^2 \frac{v^2}{c^2} sin^2\alpha \tag{19}$$

The average power $< P >$ per electron in an ensemble of electron with the same $\gamma$, but random pitch angle is

$$P = 2\sigma_T \beta^2 \gamma^2 c U_B < sin^2\alpha >= 2\sigma_T \beta^2 \gamma^2 c U_B \frac{2}{3} \tag{20}$$

The radiated power depends on the physical constants, square of the electron energy, the magnetic energy density and the pitch angle. Synchrotron emission is strongly beamed along the direction of motion which turns out to be perpendicular to the acceleration. The emission is concentrated into an angle along the direction of motion of order $1/\gamma$ . Since $1/\ \gamma$ is $<< 1$, the radiation is confined into a narrow beam of width $2/\ \gamma$ between the nulls as shown in the figure 2. But for any reasonable distribution of particles that varies smoothly with pitch angle, the elliptical components will cancel out as emission cones will be partially linearized polarized

### 4.1 Synchrotron spectrum

The beaming of the radiation has a very important effect on the observed spectrum emitted by the electron. As the electron cycles around the helical path along the magnetic field lines, any emission directed towards a distant observer is seen only when the beam is aligned with the observer's line of sight, In this case the observer sees a flash of radiation for a period which is much shorter than the gyration period. The over all spectrum of emission is the Fourier transform of the time series of pulses. Thus as a function of time the power P(t) emitted by an electron is a succession of pulses of width $\tau$ separated by $2\pi\gamma/\omega_{ce}$ ($\omega_{ce}$ is called gyro frequency) ; ie occuring with the

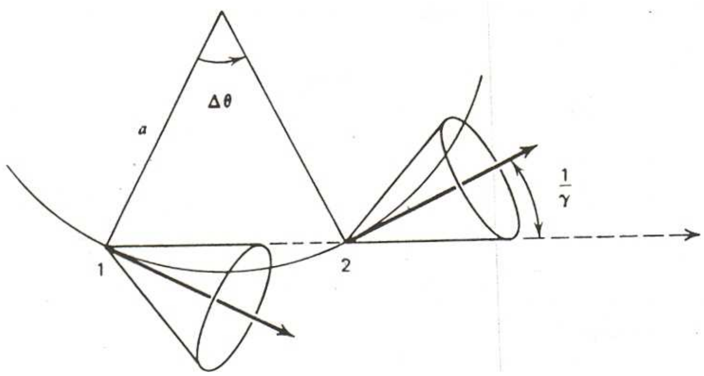


**Fig. 2.** Emission comes at various points of an accelerated particle's trajectory.

frequency $\omega_{ce}$ / 2 $\pi\gamma$ . The volume emissivity (power per unit frequency per unti volume per unit solid angle) of relativistic electrons is given by

$$\epsilon_\nu = \int_E P(\nu, E)N(E)dE \tag{21}$$

where $P(\nu, E)$ is the total power from that one electron with energy E radiates and N(E) dE is the number of electrons per unit volume and per unit solid angle moving in the direction of the observer and whose energy lies in the range of E and E+dE. The particle energy distribution can be either Maxwellian or power law. In the first case the dominant mechanism is the gyroresonance emission at discrete and low harmonic number of $\omega_{ce}$ for which emissivity can be written as a function of the density $n_e$, the magnetic field strength B and its direction relative to the observer. In the case of non- thermal power law energy distribution of electrons is of the form $AE^{-\delta}$. The gyrosynchrotron emissivity then has a power law spectrum which can be used to estimate the injected particle distribuion as well as the strength of the magnetic field B. Calculations of the emissivity for an isotropic velocity distribution of electrons in a homogeneous magnetic field have been performed by Ramaty (1969).

Let us consider the case of synchrotron emisison from N electrons with the same velocity and pitch angle. The power radiated is

$$P = \frac{dE}{dt} = \frac{4}{3}\sigma_T\beta^2\gamma^2 cU_B \tag{22}$$

at the single frequency $\nu = \gamma^2\nu_G$. The emission coefficient from an ensemble of electron is

$$\epsilon_\nu d\epsilon_\nu = \frac{dE}{dt}N(E)dE \tag{23}$$

$$Since \;\; E = \gamma m_e c^2 = \left(\frac{\nu}{\nu_G}\right)^2 m_e c^2 \tag{24}$$

The emission coefficient is

$$\epsilon_\nu = \frac{4}{3}\sigma_T \beta^2 \gamma^2 c U_B K E^{-\delta} \frac{m_e c^2 \nu^{-1/2}}{2{\nu_G}^{1/2}} \tag{25}$$

Synchrotron spectrum of a power law itself is a power law.

## 5 Summary

We have discussed a few radio emission processes in this chapter. The free - free bremsstrahlung radiation mechanism is responsible for the quiet sun radio emission. The intensity of emission depend on the atmospheric layer because of the dependence of the free-free opacity, the plasma density and the local temperature. Using typical temperature of $10^6$ K and density of $10^9 cm^{-3}$ we get the optical depth $\tau < < 1$ in the whole microwave range. At decameter wavelengths, the corona is optically thick and ray - tracing calculations are generally used to determine the temperature and density of the corona (Subramanian, 2004). In the absence of flares, the the radio emission of solar active regions is due to the free-free emission. Gyroresonance emission at the low harmonics ( s= 1, 2, 3...) is responsible for bright coronal emission from the Sun in the microwave band. Gyrosynchrotron emission is commonly observed as a broadband microwave spectrum in a typical frequency range of 2 to 20 GHz, and the spectrum of gyrosynchrotron emission peaks around 5 - 10 GHz. Most of the radio emission from solar and stellar flares are due to gyrosynchrotron emission. Synchrotron emission is important in extreme energy enviorments like extragalatic radio sources.

## References

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